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H. Algebra 2

Module 11 Review

Name: ______________________ Date: ______________ Hour: ________

11.1 Radical Expressions and Rational Exponents

Write each expression in radical form. Simplify numerical expressions when possible.

1. $64^{\frac{5}{6}}$

$2^5 = 32$

2. $(6x)^{\frac{3}{2}}$

$(\sqrt{6x})^3$

3. $(-8)^{\frac{4}{3}}$

$(-2)^4 = 16$

4. $(5r^3)^{\frac{1}{4}}$

$\sqrt[4]{5r^3}$

5. $27^{\frac{2}{3}}$

$3^2 = 9$

6. $(100a)^{\frac{1}{2}}$

$10\sqrt{a}$

7. $10^{\frac{8}{5}}$ $\sqrt[5]{10^8}$

8. $(x^2)^{\frac{2}{5}}$

$\sqrt[5]{x^4}$

9. $(7x)^{-\frac{1}{3}}$

$\sqrt[3]{\frac{1}{7x}}$

Write each expression by using rational exponents. Simplify numerical expressions when possible.

10. $(\sqrt[4]{2})^7$ $= 2^{7/4}$

11. $(\sqrt{5x})^3$ $5^{3/2}x^{3/2}$

$(5x)^{3/2}$

12. $\sqrt[5]{51^4}$ $51^{4/5}$

13. $(\sqrt{169})^3$ $169^{3/2}$

$13^3 = 2197$

14. $(\sqrt[4]{2v})^3$ $(2v)^{3/4}$

15. $(\sqrt[5]{n^2})^2$ $n^{4/5}$

16. $\frac{1}{(\sqrt{3m})^3}$ $3m^{-3/2}$

17. $\sqrt[7]{36^{14}}$ $36^{14/7} = 36^2$

1296

18. $\frac{1}{(\sqrt[5]{5p})^7}$ $(5p)^{-7/5}$

19. In every atom, electrons orbit the nucleus with a certain characteristic velocity known as the Fermi-Thomas velocity, equal to $\frac{Z^{\frac{2}{3}}}{137}c$, where Z is the number of protons in the nucleus, and c is the speed of light. In terms of c, what is the characteristic Fermi-Thomas velocity of the electrons in Uranium, for with $Z = 92$?

$\frac{92^{2/3}}{137}c = \underline{0.149c}$

11.2 Simplifying Radical Expressions

Simplify each expression. Assume all variables are positive.

1. $-3\sqrt{12r}$ $= -6\sqrt{3r}$

2. $4^{\frac{3}{2}} \cdot 4^{\frac{5}{2}}$

$4^{8/2} = 4^4 = 256$

3. $\frac{27^{\frac{4}{3}}}{27^{\frac{2}{3}}}$ $27^{2/3} = 3^2 = 9$

4. $\frac{(a^2)^2}{a^{\frac{3}{2}}b^{\frac{1}{2}} \cdot b}$ $= \frac{a^4}{a^{3/2}b^{1/2}b^{2/2}}$

$\frac{a^{5/2}}{b^{3/2}} \frac{b^{1/2}}{b^{1/2}} = \frac{a^{5/2}b^{1/2}}{b^2}$

5. $(27 \cdot 64)^{\frac{2}{3}}$

$3^2 \cdot 4^2$

$9 \cdot 16 = 144$

6. $\left(\frac{1}{243}\right)^{\frac{1}{5}}$ $= \frac{1}{3}$

7. $\frac{(25x)^{\frac{3}{2}}}{5x^{\frac{1}{2}}}$ $\frac{5^3 x^{3/2}}{5^1 x^{1/2}}$

$5^2 x = 25x$

8. $(4x)^{-\frac{1}{2}} \cdot (9x)^{\frac{1}{2}}$

$\frac{1}{2x^{1/2}} \cdot \frac{3x^{1/2}}{1} = \frac{3}{2}$

9. $3\sqrt[3]{81x^4y^2}$ $3 \cdot 3x \sqrt[3]{3xy^2}$

$9x\sqrt[3]{3xy^2}$

10. $-5\sqrt[3]{-500x^5y^3}$

$-25xy\sqrt[3]{-4x^2}$

11. $\sqrt[4]{32x^7y^{12}z}$

$2xy^3\sqrt[4]{2x^3z}$

12. $-\sqrt{75a^3b^6}$

$-5ab^3\sqrt{5a}$

13. The frequency, f, in Hz, at which a simple pendulum rocks back and forth is given by $f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$, where g is the strength of the gravitational field at the location of the pendulum, and l is the length of the pendulum.

a) Rewrite the formula so that it gives the length l of the pendulum in terms of g and f. Then simplify the formula using the fact that the gravitational field is approximately 32 ft/s².

$f = \frac{1}{2\pi}\sqrt{\frac{g}{l}} \rightarrow 2\pi f = \sqrt{\frac{g}{l}}$ $l \frac{g}{l} = (2\pi f)^2 l$

$\frac{l(2\pi f)^2}{(2\pi f)^2} = \frac{g}{2\pi f^2}$ $l = \frac{g}{(2\pi f)^2}$ $l = \frac{32}{(2\pi f)^2}$ $\frac{32}{(2\pi)^2 f^2}$ $l = .81/f^2$

b) Use the equation found in part a) to find the length of a pendulum, to the nearest foot, that has a frequency of 0.52 Hz.

$l = \frac{.81}{f^2}$ $l = \frac{.81}{(.52)^2} =$

$l = 3$ ft

11.3 Solving Radical Equations

Solve each equation. Check your answers.

1. $\sqrt{x+6}^{2} = 7^{2}$

$x+6=49$

$x=43$

2. $\sqrt{5x}^{2} = 10^{2}$

$5x=100$

$x=20$

3. $\sqrt{2x+5} = \sqrt{3x-1}$

$2x+5=3x-1$

$6=x$ ✓

4. $\sqrt{x+4}^{2} = (3\sqrt{x})^{2}$

$x+4=9x$

$-x \quad -x$

$\frac{4}{8}=\frac{8x}{8}$

$x=\frac{1}{2}$ ✓

5. $\sqrt[3]{x-6}^{3} = \sqrt[3]{3x+24}^{3}$

$x-6=3x+24$

$-30=2x$

$x=-15$

6. $(3\sqrt[3]{x})^{3} = (\sqrt[3]{7x+5})^{3}$

$27x=7x+5$

$20x=5$

$x=\frac{1}{4}$

7. $\sqrt{-14x+2}^{2} = (x-3)^{2}$

$-14x+2=x^2-6x+9$

$+14x-2 \quad +14x-2$

$0=x^2+8x+7$

$0=(x+7)(x+1)$

~~$x=-7$~~ ~~$x=-1$~~

no solutions

8. $(x+4)^{\frac{1}{2}} = 6$

$x+4=36$

$x=32$

9. $\frac{4(x-3)^{\frac{1}{2}}}{4} = \frac{8}{4}$

$\left((x-3)^{1/2}\right)^2 = (2)^2$

$x-3=4$

$x=7$ ✓

10. $\frac{4(x-12)^{\frac{1}{3}}}{4} = \frac{-16}{4}$

$(x-12)^{1/3}=-4$

$x-12=-64$
$+12 \quad +12$

$x=-52$ ✓

11. $\sqrt{3x+6}^{\,2} = 3^2$

$3x+6=9$

$3x=3$

$x=1$

12. $\sqrt{x-4}+3=9$
$-3 \quad -3$

$\sqrt{x-4}^{\,2} = 6^2$

$x-4=36$

$x=40$

13. $\sqrt{x+7}=\sqrt{2x-1}$

$x+7=2x-1$

$7=x-1$

$x=8$

14. $\sqrt{2x-7}^{\,2}=2x^2$

$2x-7=4x^2$
$-2x+7 \quad -2x+7$

$0=4x^2-2x+7$

$x=\frac{-2 \pm \sqrt{(-2)^2-4(4)7}}{8}$

$2 \pm \sqrt{-108}$

No Solutions

15. A biologist is studying two species of animals in a habitat. The population, p_1, of one of the species is growing according to $p_1 = 500t^{\frac{3}{2}}$, and the population, p_2, of the other species is growing according to $p_2 = 100t^2$, where time, t, is measured in years. After how many years will the populations of the two species be equal?

$\frac{500t^{3/2}}{100} = \frac{100t^2}{100}$

$\frac{5t^{3/2}}{t^{3/2}} = \frac{t^2}{t^{3/2}}$

$(5)^2 = (t^{1/2})^2$

$25=t$